Mass determination from Constraint Effective Potential

A. Agodi
G. Andronico
Dipartimento di Fisica dell'Universitá di Catania
INFN Sezione di Catania

The Constraint Effective Potential (CEP) allows a determination of the mass and other quantities directly, without relying upon asymptotic correlator decays. We report and discuss the results of some mass calculations in $\left(\lambda\Phi^4\right)_4$, obtained from CEP and our improved version of CEP (ICEP).

1. Introduction

It has been shown that an Improved version (ICEP) of the Constraint Effective Potential (CEP)[1] reduces finite size effects in $\left(\phi^4\right)_4$ lattice calculations[2]. The Constraint Effective Potential $U\left(\Omega,\overline{\phi}\right)$ (where Ω is the lattice 4-volume and $\overline{\phi}$ the VEV of the field) was defined as

$$\exp\left(-\Omega U\left(\Omega,\overline{\phi}\right)\right) = \int D\phi\,\delta\left(M\left[\phi\right]-\overline{\phi}\right)\exp\left(-S\left[\phi\right]\right) \tag{1}$$

being $M\left[\phi\right]=\frac{1}{\Omega}\int \mathrm{d}^d x\phi\left(x\right)$. With the function $W\left(\Omega,j\right)$ of the external source j, defined by

$$\exp\left(\Omega W\left(\Omega,j\right)\right) = \int d\overline{\phi}\exp\left[\Omega\left(j\overline{\phi}-U\left(\Omega,\overline{\phi}\right)\right)\right], \tag{2}$$

the effective potential Γ is the Legendre transform

$$\Gamma\left(\Omega,\overline{\phi}\right)=\sup_j\left(j\overline{\phi}-W\left(\Omega,j\right)\right).$$

It has been shown that

$$\lim_{\Omega\to\infty}U\left(\Omega,\overline{\phi}\right)=\lim_{\Omega\to\infty}\Gamma\left(\Omega,\overline{\phi}\right).$$

For big enough Ω

$$\Gamma\left(\Omega,\overline{\phi}\right)\approx U\left(\Omega,\overline{\phi}\right) \tag{3}$$

We have shown[2] that better results for the values of

$$J=\frac{\partial\Gamma\left(\Omega,\overline{\phi}\right)}{\partial\overline{\phi}}$$

are obtained by evaluating (2) with the saddle point method. In this way we get

$$\Gamma\left(\Omega,\overline{\phi}\right)=U\left(\Omega,\overline{\phi}\right)+\frac{1}{2\Omega}\ln U''\left(\Omega,\overline{\phi}\right)+K\left(\Omega\right) \tag{4}$$

where $K\left(\Omega\right)$ is $\overline{\phi}$-independent and

$$\lim_{\Omega\to\infty}K\left(\Omega\right)=0.$$

This is what we call Improved CEP (ICEP).

In the present work we present some preliminary results, as obtained from the behavior of

$$\Gamma'=\frac{\partial\Gamma\left(\Omega,\overline{\phi}\right)}{\partial\overline{\phi}}$$

and

$$\Gamma''=\frac{\partial^2\Gamma\left(\Omega,\overline{\phi}\right)}{\partial\overline{\phi}^2}$$

on a 16^4 lattice.

2. CEP

From the assumption (3) whose reliability was checked in [2] it follows

$$\Gamma'' = U''\left(\Omega,\varphi\right) = \left\langle V''\right\rangle_{\overline{\phi}}-\Omega\left\langle\left(V'-\left\langle V'\right\rangle_{\overline{\phi}}\right)^2\right\rangle_{\overline{\phi}}$$

where $V'=r_0\phi+\lambda_0\phi^3$, $V''=r_0+3\lambda_0\phi^2$, $\left\langle\bullet\right\rangle_{\overline{\phi}}$ means averaging on the ensemble with $\overline{\phi}=\left\langle\phi\right\rangle$ fixed, r_0 and λ_0 are, respectively, the quadratic and quartic coupling.

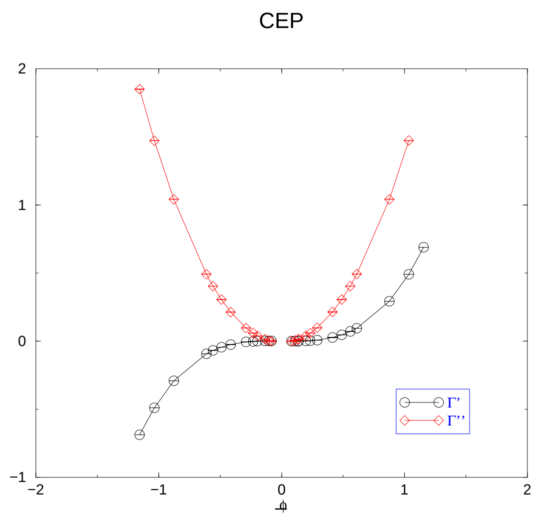

Figure 1. Results for Γ' and Γ'' as obtained from CEP

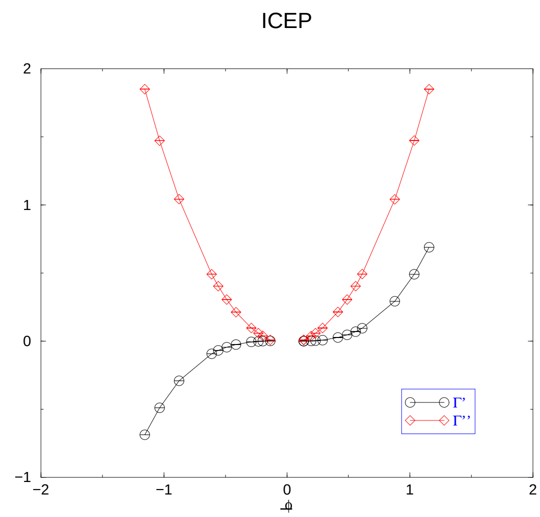

Figure 2. Results for Γ' and Γ'' as obtained from ICEP

3. ICEP

From eq. (4) it follows

$$\begin{aligned}\Gamma'' &= U''(\Omega,\varphi) \\ &\quad + \frac{1}{2\Omega}\left[\frac{U^{iv}(\Omega,\varphi)}{U''(\Omega,\varphi)} - \left(\frac{U'''(\Omega,\varphi)}{U''(\Omega,\varphi)}\right)^2\right]\end{aligned}$$

.

The $U(\Omega,\varphi)$ derivatives involved above are obtained in a simpler way by suitably exploiting [2] eq (1).

4. Results

We have determined Γ' and Γ'' as functions of $\overline{\phi}$ for $\lambda_0 = 0.5$, $\mathrm{r}_0 = -0.2279$ (near the critical value),$\mathrm{r}_0 = -0.2179$ (symmetric domain) and $\mathrm{r}_0 = -0.2379$ (broken symmetry domain).

With φ satisfying $\Gamma'(\varphi) = 0$ one has, by definition, $\Gamma''(\varphi) = m^2$. From our data it turns out that, for r_0 in the symmetric domain $m^2 = 0$. Near the critical value m^2 is consistent with a vanishing value. For r_0 in the broken symmetry domain, Fig. 1 shows the CEP results and Fig. 2 those from ICEP. From these data we get

	φ	m^2
CEP	-0.1485 ± 0.0005	0.0177 ± 0.0002
	0.1542 ± 0.0015	0.0200 ± 0.0006
ICEP	-0.154 ± 0.001	0.0163 ± 0.0004
	0.155 ± 0.002	0.0173 ± 0.0008

The ICEP results are symmetric while the CEP are not. This might be due to ICEP reducing finite size effects.

REFERENCES

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